Pearson Edexcel Level 3 GCE

Practice Paper 1

Time: 1 hour 30 minutes | Paper Reference **9FM0/4D**

Further Mathematics
Advanced
Paper 4D: Decision Mathematics 2

You must have:
calculator

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 76. There are 8 questions.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. (a) Find the general solution of the recurrence relation $u_{n+2} - 6u_{n+1} + 9u_n = 0$.

(3)

Given that $u_1 = \frac{21}{4}$ and $u_2 = -\frac{93}{4}$,

(b) find the particular solution of the recurrence relation $u_{n+2} - 6u_{n+1} + 9u_n = 15$.

(6)

(Total for Question 1 is 9 marks)

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2. The pay-off matrix for a zero-sum game between X and Y is shown below.

	***Y* plays 1**	***Y* plays 2**	***Y* plays 3**
***X* plays 1**	3	2	1
***X* plays 2**	1	4	2
***X* plays 3**	−2	2	1
***X* plays 4**	1	5	4

(a) Use dominance arguments to reduce the pay-off matrix so that one player has only two choices.

(2)

(b) Verify that X and Y should play mixed strategies.

(2)

(c) Use your reduced pay-off matrix to find the optimum mixed strategy for the player with two choices and state the value of the game to her.

(7)

(Total for Question 2 is 11 marks)

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3. Four workers P, Q, R and S are to be assigned to four tasks W, X, Y and Z. Each worker can only be assigned to one task and each task is to be completed by just one worker.

The table shows the cost, in £s, of allocating each worker to each task.

	W	***X***	***Y***	***Z***
P	35	41	52	47
Q	43	45	48	46
R	37	48	45	50
S	42	50	44	48

The Hungarian algorithm is to be used to allocate workers to tasks so that the total cost is minimised.

(a) Reducing rows first, use the Hungarian algorithm to find the optimum allocation showing the table used at each stage. State the minimum cost.

(7)

(b) Formulate the problem as a linear programming problem. You should define your decision variables, objective function and constraints.

(5)

(Total for Question 3 is 12 marks)

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4. Three warehouses A, B and C supply three shops P, Q and R with pallets of frozen cakes. The table shows the transportation costs, in £s per pallet, from each warehouse to each shop. The table also shows the supply available at each warehouse and the demand required at each shop.

	P	***Q***	***R***	**Supply**
A	11	12	15	10
B	14	10	14	16
C	12	16	13	14
Demand	12	14	14	

(a) Use the north-west corner method to find an initial solution.

(1)

(b) Use the transportation algorithm to solve the problem, making your method clear. Explain how you know that your solution is optimal.

(11)

(Total for Question 4 is 12 marks)

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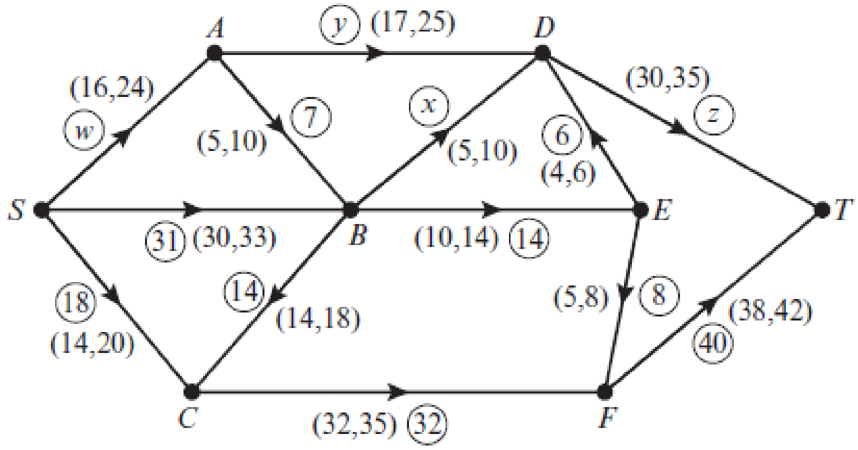
5. The diagram shows a capacitated directed network. The numbers on each arc represent lower and upper capacities. The numbers in circles represent an initial flow.



(a) Find the values of w, x, y and z.

(3)

(b) Find the value of the cut through

(i) AD, BD, BE, CF

(ii) DT, DE, BE, CF

(2)

(c) Explain what your answers to part (b) show about the maximum flow.

(1)

(d) Find two flow-augmenting paths to increase the flow by a total of 4.

(4)

(e) Show that the flow is then maximal.

(2)

(Total for Question 5 is 12 marks)

6. A small start-up company can choose between three projects A, B and C. It is estimated that:

- Project A has a probability of $\frac{1}{3}$ of making a £20 000 profit, a probability of $\frac{1}{2}$ of making a £10 000 profit and a probability of $\frac{1}{6}$ of making a £20 000 loss.

- Project B has a probability of $\frac{1}{4}$ of making a £15 000 profit, a probability of $\frac{1}{2}$ of making an £8000 profit and a probability of $\frac{1}{4}$ of making a £2000 loss.

- Project C has a probability of $\frac{1}{2}$ of making a £12 000 profit, a probability of $\frac{1}{4}$ of making a £5000 profit and a probability of $\frac{1}{4}$ of making a £1000 loss.

(a) Draw a decision tree to represent the decisions and possible pay-offs. Use the expected monetary value (EMV) criterion to determine which project should be undertaken.

(3)

(b) Explain why it would be advisable to use a utility function in this case.

(1)

(c) Draw a new decision tree using the utility function $u(x) = \sqrt{(x + 25)}$, where x is the profit in £1000s.

(3)

(d) Determine which project should be undertaken using expected utility as the criterion.

(1)

(Total for Question 6 is 8 marks)

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7. A company builds trailers and has overhead costs amounting to £10 000 per month for building up to 14 trailers. More than 14 trailers may be built in a month, for an additional overhead cost of £800.

The maximum number of trailers that can be built in any month is 17. Up to 2 trailers can be stored in any month for a cost of £150 each.

Trailers are delivered at the end of each month. There are no trailers in stock at the beginning of July and there must be none left over at the end of October.

The order book for trailers is:

Month	July	August	September	October
Number ordered	15	13	17	15

Use dynamic programming to produce a schedule that minimises the production costs.

(The total for Question 7 is 12 marks)

TOTAL FOR DECISION MATHEMATICS 2 IS 76 MARKS